

Notes: Exponential Functions: Finance Problems  
(you will need a calculator)

*Compound interest* is interest paid on an initial investment, called the *principal*, and on previously earned interest. Interest earned is often expressed as an *annual percent*, but the interest is usually compounded more than once per year. So, the exponential growth model  $y = a(1 + r)^t$  must be modified for compound interest problems.

## Core Concept

### Compound Interest

Consider an initial principal  $P$  deposited in an account that pays interest at an annual rate  $r$  (expressed as a decimal), compounded  $n$  times per year. The amount  $A$  in the account after  $t$  years is given by

$$A = P\left(1 + \frac{r}{n}\right)^{nt}.$$

### EXAMPLE 5 Finding the Balance in an Account

You deposit \$9000 in an account that pays 1.46% annual interest. Find the balance after 3 years when the interest is compounded quarterly.

#### SOLUTION

With interest compounded quarterly (4 times per year), the balance after 3 years is

$$\begin{aligned} A &= P\left(1 + \frac{r}{n}\right)^{nt} && \text{Write compound interest formula.} \\ &= 9000\left(1 + \frac{0.0146}{4}\right)^{4 \cdot 3} && P = 9000, r = 0.0146, n = 4, t = 3 \\ &\approx 9402.21. && \text{Use a calculator.} \end{aligned}$$

► The balance at the end of 3 years is \$9402.21.

Example 6: You borrow \$189,000 to buy a house and the current interest rate is 4.5% compounded annually.

a) If you take out a 15-year loan, how much will you owe? How much will you pay per month?

b) If you take out a 30-year loan, how much will you owe? How much will you pay per month?

## Solving Real-Life Problems

You have learned that the balance of an account earning compound interest is given by  $A = P\left(1 + \frac{r}{n}\right)^{nt}$ . As the frequency  $n$  of compounding approaches positive infinity, the compound interest formula approximates the following formula.

### Core Concept

#### Continuously Compounded Interest

When interest is compounded *continuously*, the amount  $A$  in an account after  $t$  years is given by the formula

$$A = Pe^{rt}$$

where  $P$  is the principal and  $r$  is the annual interest rate expressed as a decimal.

Example 7: You deposit \$5000 in an account that pays 2.25% annual interest. Find the balance after 5 years when the interest is compounded continuously.

Find the balance when the interest is compounded daily.

Example 8:

When you borrow money, do you want a higher interest rate or a lower interest rate? Why?

When you invest money, do you want a higher interest rate or a lower interest rate? Why?

Write the equation you use to answer the question. For number 38, fill in the table with your answer. Show the equation you used to find each answer in the table.

37. **PROBLEM SOLVING** You deposit \$5000 in an account that pays 2.25% annual interest. Find the balance after 5 years when the interest is compounded quarterly. (See Example 5.)

38. **DRAWING CONCLUSIONS** You deposit \$2200 into three separate bank accounts that each pay 3% annual interest. How much interest does each account earn after 6 years?

Account	Compounding	Balance after 6 years
1	quarterly	
2	monthly	
3	daily	

Use the information in #38 to find the balance after 6 years if the interest is compounded continuously.

Read each problem carefully before you try to analyze the work.

39. **ERROR ANALYSIS** You invest \$500 in the stock of a company. The value of the stock decreases 2% each year. Describe and correct the error in writing a model for the value of the stock after  $t$  years.

**X**

$$y = \left( \begin{array}{l} \text{Initial} \\ \text{amount} \end{array} \right) \left( \begin{array}{l} \text{Decay} \\ \text{factor} \end{array} \right)^t$$

$$y = 500(0.02)^t$$

40. **ERROR ANALYSIS** You deposit \$250 in an account that pays 1.25% annual interest. Describe and correct the error in finding the balance after 3 years when the interest is compounded quarterly.

**X**

$$A = 250 \left( 1 + \frac{1.25}{4} \right)^{4 \cdot 3}$$

$$A = \$6533.29$$

Show each equation you used to answer each question. Use the space provided.

**In Exercises 41–44, use the given information to find the amount  $A$  in the account earning compound interest after 6 years when the principal is \$3500.**

41.  $r = 2.16\%$ , compounded quarterly
42.  $r = 2.29\%$ , compounded monthly
43.  $r = 1.83\%$ , compounded daily
44.  $r = 1.26\%$ , compounded monthly
45.  $r = 2.75\%$ , compounded continuously

Review:

The following problems are NOT finance problems. Read each problem carefully.

21. **MODELING WITH MATHEMATICS** The value of a mountain bike  $y$  (in dollars) can be approximated by the model  $y = 200(0.75)^t$ , where  $t$  is the number of years since the bike was new. (*See Example 2.*)
  - a. Tell whether the model represents exponential growth or exponential decay.
  - b. Identify the annual percent increase or decrease in the value of the bike.
  - c. Estimate when the value of the bike will be \$50.
  
24. **MODELING WITH MATHEMATICS** You take a 325 milligram dosage of ibuprofen. During each subsequent hour, the amount of medication in your bloodstream decreases by about 29% each hour.
  - a. Write an exponential decay model giving the amount  $y$  (in milligrams) of ibuprofen in your bloodstream  $t$  hours after the initial dose.
  - b. Estimate how long it takes for you to have 100 milligrams of ibuprofen in your bloodstream.

Match the function to its graph by identifying the given function as a growth or decay. Then identify and use the “starting points” to help you match the correct graphs. DO NOT CONNECT THE FUNCTIONS AND GRAPHS WITH LINES.

a.  $f(x) = 2^x$

b.  $f(x) = 3^x$

c.  $f(x) = 4^x$

d.  $f(x) = \left(\frac{1}{2}\right)^x$

e.  $f(x) = \left(\frac{1}{3}\right)^x$

f.  $f(x) = \left(\frac{1}{4}\right)^x$

