

# 5.3 Exercises

## Vocabulary and Core Concept Check

- WRITING** Explain the Remainder Theorem in your own words. Use an example in your explanation.
- VOCABULARY** What form must the divisor have to make synthetic division an appropriate method for dividing a polynomial? Provide examples to support your claim.
- VOCABULARY** Write the polynomial divisor, dividend, and quotient functions represented by the synthetic division shown at the right.
- WRITING** Explain what the colored numbers represent in the synthetic division in Exercise 3.

$$\begin{array}{r|rrrr}
 -3 & 1 & -2 & -9 & 18 \\
 & & -3 & 15 & -18 \\
 \hline
 & 1 & -5 & 6 & 0
 \end{array}$$

## Monitoring Progress and Modeling with Mathematics

In Exercises 5–10, divide using polynomial long division. (See Example 1.)

- $(x^2 + x - 17) \div (x - 4)$
- $(3x^2 - 14x - 5) \div (x - 5)$
- $(x^3 + x^2 + x + 2) \div (x^2 - 1)$
- $(7x^3 + x^2 + x) \div (x^2 + 1)$
- $(5x^4 - 2x^3 - 7x^2 - 39) \div (x^2 + 2x - 4)$
- $(4x^4 + 5x - 4) \div (x^2 - 3x - 2)$


In Exercises 11–18, divide using synthetic division. (See Examples 2 and 3.)

- $(x^2 + 8x + 1) \div (x - 4)$
- $(4x^2 - 13x - 5) \div (x - 2)$
- $(2x^2 - x + 7) \div (x + 5)$
- $(x^3 - 4x + 6) \div (x + 3)$
- $(x^2 + 9) \div (x - 3)$
- $(3x^3 - 5x^2 - 2) \div (x - 1)$
- $(x^4 - 5x^3 - 8x^2 + 13x - 12) \div (x - 6)$
- $(x^4 + 4x^3 + 16x - 35) \div (x + 5)$


**ANALYZING RELATIONSHIPS** In Exercises 19–22, match the equivalent expressions. Justify your answers.

- $(x^2 + x - 3) \div (x - 2)$
  - $(x^2 - x - 3) \div (x - 2)$
  - $(x^2 - x + 3) \div (x - 2)$
  - $(x^2 + x + 3) \div (x - 2)$
- A.  $x + 1 - \frac{1}{x - 2}$       B.  $x + 3 + \frac{9}{x - 2}$   
 C.  $x + 1 + \frac{5}{x - 2}$       D.  $x + 3 + \frac{3}{x - 2}$

**ERROR ANALYSIS** In Exercises 23 and 24, describe and correct the error in using synthetic division to divide  $x^3 - 5x + 3$  by  $x - 2$ .

23.  
$$\begin{array}{r|rrrr}
 2 & 1 & 0 & -5 & 3 \\
 & & 2 & 4 & -2 \\
 \hline
 & 1 & 2 & -1 & 1
 \end{array}$$

$$\frac{x^3 - 5x + 3}{x - 2} = x^3 + 2x^2 - x + 1$$

24.  
$$\begin{array}{r|rrr}
 2 & 1 & -5 & 3 \\
 & & 2 & -6 \\
 \hline
 & 1 & -3 & -3
 \end{array}$$

$$\frac{x^3 - 5x + 3}{x - 2} = x^2 - 3x - \frac{3}{x - 2}$$

In Exercises 25–32, use synthetic division to evaluate the function for the indicated value of  $x$ . (See Example 4.)

25.  $f(x) = -x^2 - 8x + 30; x = -1$

26.  $f(x) = 3x^2 + 2x - 20; x = 3$

27.  $f(x) = x^3 - 2x^2 + 4x + 3; x = 2$

28.  $f(x) = x^3 + x^2 - 3x + 9; x = -4$

29.  $f(x) = x^3 - 6x + 1; x = 6$

30.  $f(x) = x^3 - 9x - 7; x = 10$

31.  $f(x) = x^4 + 6x^2 - 7x + 1; x = 3$

32.  $f(x) = -x^4 - x^3 - 2; x = 5$

33. **MAKING AN ARGUMENT** You use synthetic division to divide  $f(x)$  by  $(x - a)$  and find that the remainder equals 15. Your friend concludes that  $f(15) = a$ . Is your friend correct? Explain your reasoning.

34. **THOUGHT PROVOKING** A polygon has an area represented by  $A = 4x^2 + 8x + 4$ . The figure has at least one dimension equal to  $2x + 2$ . Draw the figure and label its dimensions.

35. **USING TOOLS** The total attendance  $A$  (in thousands) at NCAA women's basketball games and the number  $T$  of NCAA women's basketball teams over a period of time can be modeled by

$$A = -1.95x^3 + 70.1x^2 - 188x + 2150$$

$$T = 14.8x + 725$$

where  $x$  is in years and  $0 < x < 18$ . Write a function for the average attendance per team over this period of time.

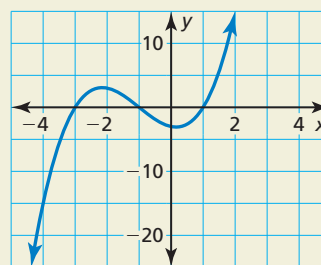


36. **COMPARING METHODS** The profit  $P$  (in millions of dollars) for a DVD manufacturer can be modeled by  $P = -6x^3 + 72x$ , where  $x$  is the number (in millions) of DVDs produced. Use synthetic division to show that the company yields a profit of \$96 million when 2 million DVDs are produced. Is there an easier method? Explain.

37. **CRITICAL THINKING** What is the value of  $k$  such that  $(x^3 - x^2 + kx - 30) \div (x - 5)$  has a remainder of zero?

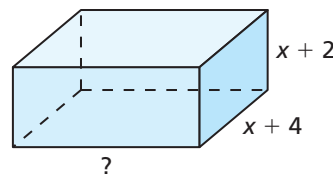
- (A) -14                      (B) -2  
(C) 26                        (D) 32

38. **HOW DO YOU SEE IT?** The graph represents the polynomial function  $f(x) = x^3 + 3x^2 - x - 3$ .



- a. The expression  $f(x) \div (x - k)$  has a remainder of  $-15$ . What is the value of  $k$ ?
- b. Use the graph to compare the remainders of  $(x^3 + 3x^2 - x - 3) \div (x + 3)$  and  $(x^3 + 3x^2 - x - 3) \div (x + 1)$ .

39. **MATHEMATICAL CONNECTIONS** The volume  $V$  of the rectangular prism is given by  $V = 2x^3 + 17x^2 + 46x + 40$ . Find an expression for the missing dimension.



40. **USING STRUCTURE** You divide two polynomials and obtain the result  $5x^2 - 13x + 47 - \frac{102}{x + 2}$ . What is the dividend? How did you find it?

## Maintaining Mathematical Proficiency Reviewing what you learned in previous grades and lessons

Find the zero(s) of the function. (Sections 4.1 and 4.2)

41.  $f(x) = x^2 - 6x + 9$

42.  $g(x) = 3(x + 6)(x - 2)$

43.  $g(x) = x^2 + 14x + 49$

44.  $h(x) = 4x^2 + 36$