Vocabulary and Core Concept Check

1. **WRITING** Describe three different methods to expand $(x + 3)^3$.

2. **WRITING** Is $(a + b)(a - b) = a^2 - b^2$ an identity? Explain your reasoning.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, find the sum. *(See Example 1.)*

3. $(3x^2 + 4x - 1) + (-2x^2 - 3x + 2)$

4. $(-5x^2 + 4x - 2) + (-8x^2 + 2x + 1)$

5. $(12x^5 - 3x^4 + 2x - 5) + (8x^4 - 3x^3 + 4x + 1)$

6. $(8x^4 + 2x^2 - 1) + (3x^3 - 5x^2 + 7x + 1)$

7. $(7x^6 + 2x^5 - 3x^2 + 9x) + (5x^5 + 8x^3 - 6x^2 + 2x - 5)$

8. $(9x^4 - 3x^3 + 4x^2 + 5x + 7) + (11x^4 - 4x^2 - 11x - 9)$

In Exercises 9–14, find the difference. *(See Example 2.)*

9. $(3x^3 - 2x^2 + 4x - 8) - (5x^3 + 12x^2 - 3x - 4)$

10. $(7x^4 - 9x^3 - 4x^2 + 5x + 6) - (2x^4 + 3x^3 - x^2 + x - 4)$

11. $(5x^6 - 2x^4 + 9x^3 + 2x - 4) - (7x^5 - 8x^4 + 2x - 11)$

12. $(4x^5 - 7x^3 - 9x^2 + 18) - (14x^5 - 8x^4 + 11x^2 + x)$

13. $(8x^5 + 6x^3 - 2x^2 + 10x) - (9x^5 - x^3 - 13x^2 + 4)$

14. $(11x^4 - 9x^2 + 3x + 11) - (2x^4 + 6x^3 + 2x - 9)$

15. **MODELING WITH MATHEMATICS** During a recent period of time, the numbers (in thousands) of males M and females F that attend degree-granting institutions in the United States can be modeled by

$M = 19.7t^2 + 310.5t + 7539.6$

$F = 28t^2 + 368t + 10127.8$

where t is time in years. Write a polynomial to model the total number of people attending degree-granting institutions. Interpret its constant term.

16. **MODELING WITH MATHEMATICS** A farmer plants a garden that contains corn and pumpkins. The total area (in square feet) of the garden is modeled by the expression $2x^2 + 5x + 4$. The area of the corn is modeled by the expression $x^2 - 3x + 2$. Write an expression that models the area of the pumpkins.

In Exercises 17–24, find the product. *(See Example 3.)*

17. $7x^3(5x^2 + 3x + 1)$

18. $-4x^5(11x^3 + 2x^2 + 9x + 1)$

19. $(5x^2 - 4x + 6)(-2x + 3)$

20. $(-x - 3)(2x^2 + 5x + 8)$

21. $(x^2 - 2x - 4)(x^2 - 3x - 5)$

22. $(3x^2 + x - 2)(-4x^2 - 2x - 1)$

23. $(3x^3 - 9x + 7)(x^2 - 2x + 1)$

24. $(4x^2 - 8x - 2)(x^4 + 3x^2 + 4x)$

ERROR ANALYSIS In Exercises 25 and 26, describe and correct the error in performing the operation.

25.

$(x^2 - 3x + 4) - (x^3 + 7x - 2)$

$= x^2 - 3x + 4 - x^3 + 7x - 2$

$= -x^3 + x^2 + 4x + 2$

26.

$(2x - 7)^3 = (2x)^3 - 7^3$

$= 8x^3 - 343$

In Exercises 27–32, find the product of the binomials.
(See Example 4.)

27. $(x - 3)(x + 2)(x + 4)$

28. $(x - 5)(x + 2)(x - 6)$

29. $(x - 2)(3x + 1)(4x - 3)$

30. $(2x + 5)(x - 2)(3x + 4)$

31. $(3x - 4)(5 - 2x)(4x + 1)$

32. $(4 - 5x)(1 - 2x)(3x + 2)$

33. REASONING Prove the polynomial identity $(a + b)(a - b) = a^2 - b^2$. Then give an example of two whole numbers greater than 10 that can be multiplied using mental math and the given identity. Justify your answers. *(See Example 5.)*

34. NUMBER SENSE You have been asked to order textbooks for your class. You need to order 29 textbooks that cost $31 each. Explain how you can use the polynomial identity $(a + b)(a - b) = a^2 - b^2$ and mental math to find the total cost of the textbooks.
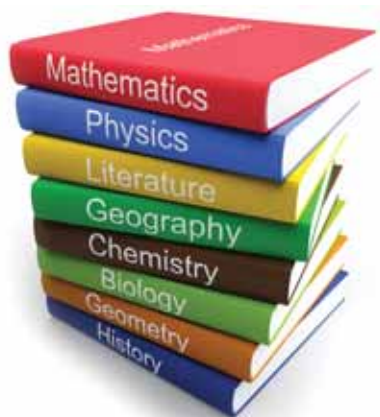
**In Exercises 35–42, find the product.** *(See Example 6.)*

35. $(x - 9)(x + 9)$ **36.** $(m + 6)^2$

37. $(3c - 5)^2$ **38.** $(2y - 5)(2y + 5)$

39. $(7h + 4)^2$ **40.** $(9g - 4)^2$

41. $(2k + 6)^3$ **42.** $(4n - 3)^3$

In Exercises 43–48, use Pascal's Triangle to expand the binomial. *(See Example 7.)*

43. $(2t + 4)^3$ **44.** $(6m + 2)^2$

45. $(2q - 3)^4$ **46.** $(g + 2)^5$

47. $(yz + 1)^5$ **48.** $(np - 1)^4$

49. COMPARING METHODS Find the product of the expression $(a^2 + 4b^2)^2(3a^2 - b^2)^2$ using two different methods. Which method do you prefer? Explain.
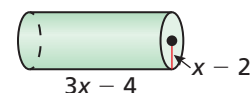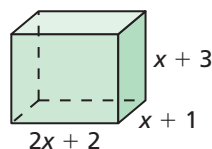
50. THOUGHT PROVOKING Adjoin one or more polygons to the rectangle to form a single new polygon whose perimeter is double that of the rectangle. Find the perimeter of the new polygon.
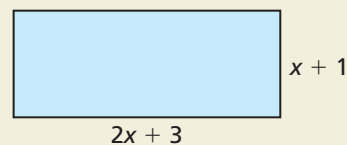
MATHEMATICAL CONNECTIONS In Exercises 51 and 52, write an expression for the volume of the figure as a polynomial in standard form.

51. $V = \ell w h$ **52.** $V = \pi r^2 h$

53. MODELING WITH MATHEMATICS Two people make three deposits into their bank accounts earning the same simple interest rate r.

Person A		Account No. 2-5384100608
Date	Transaction	Amount
01/01/2012	Deposit	$2000.00
01/01/2013	Deposit	$3000.00
01/01/2014	Deposit	$1000.00

| Person B		Account No. 1-5233032905
Date	Transaction	Amount
01/01/2012	Deposit	$5000.00
01/01/2013	Deposit	$1000.00
01/01/2014	Deposit	$4000.00

Person A's account is worth

$$2000(1 + r)^3 + 3000(1 + r)^2 + 1000(1 + r)$$

on January 1, 2015.

a. Write a polynomial for the value of Person B's account on January 1, 2015.

b. Write the total value of the two accounts as a polynomial in standard form. Then interpret the coefficients of the polynomial.

c. Suppose their interest rate is 0.05. What is the total value of the two accounts on January 1, 2015?

54. Find an expression for the volume of the cube outside the sphere.
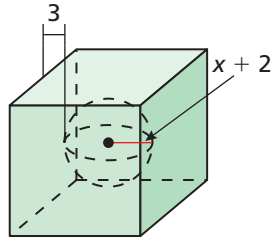
55. MAKING AN ARGUMENT Your friend claims the sum of two binomials is always a binomial and the product of two binomials is always a trinomial. Is your friend correct? Explain your reasoning.

56. HOW DO YOU SEE IT? You make a tin box by cutting x-inch-by-x-inch pieces of tin off the corners of a rectangle and folding up each side. The plan for your box is shown.
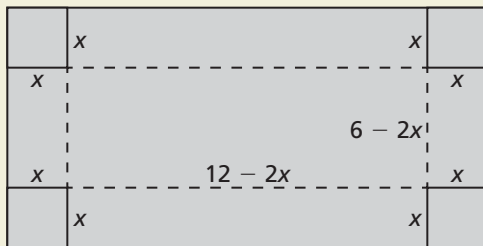
   a. What are the dimensions of the original piece of tin?

   b. Write a function that represents the volume of the box. Without multiplying, determine its degree.

USING TOOLS In Exercises 57–60, use a graphing calculator to make a conjecture about whether the two functions are equivalent. Explain your reasoning.

57. $f(x) = (2x - 3)^3$; $g(x) = 8x^3 - 36x^2 + 54x - 27$

58. $h(x) = (x + 2)^5$;
$k(x) = x^5 + 10x^4 + 40x^3 + 80x^2 + 64x$

59. $f(x) = (-x - 3)^4$;
$g(x) = x^4 + 12x^3 + 54x^2 + 108x + 80$

**60.** $f(x) = (-x + 5)^3$; $g(x) = -x^3 + 15x^2 - 75x + 125$

61. REASONING Copy Pascal's Triangle and add rows for $n = 6, 7, 8, 9$, and 10. Use the new rows to expand $(x + 3)^7$ and $(x - 5)^9$.

62. ABSTRACT REASONING You are given the function $f(x) = (x + a)(x + b)(x + c)(x + d)$. When $f(x)$ is written in standard form, show that the coefficient of x^3 is the sum of a, b, c, and d, and the constant term is the product of a, b, c, and d.

63. DRAWING CONCLUSIONS Let $g(x) = 12x^4 + 8x + 9$ and $h(x) = 3x^5 + 2x^3 - 7x + 4$.

   a. What is the degree of the polynomial $g(x) + h(x)$?

   b. What is the degree of the polynomial $g(x) - h(x)$?

   c. What is the degree of the polynomial $g(x) \cdot h(x)$?

   d. In general, if $g(x)$ and $h(x)$ are polynomials such that $g(x)$ has degree m and $h(x)$ has degree n, and $m > n$, what are the degrees of $g(x) + h(x)$, $g(x) - h(x)$, and $g(x) \cdot h(x)$?

64. FINDING A PATTERN In this exercise, you will explore the sequence of square numbers. The first four square numbers are represented below.
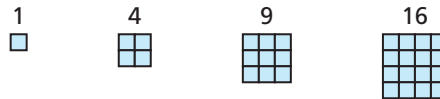
   a. Find the differences between consecutive square numbers. Explain what you notice.

   b. Show how the polynomial identity $(n + 1)^2 - n^2 = 2n + 1$ models the differences between square numbers.

   c. Prove the polynomial identity in part (b).

65. CRITICAL THINKING Recall that a Pythagorean triple is a set of positive integers a, b, and c such that $a^2 + b^2 = c^2$. The numbers 3, 4, and 5 form a Pythagorean triple because $3^2 + 4^2 = 5^2$. You can use the polynomial identity $(x^2 - y^2)^2 + (2xy)^2 = (x^2 + y^2)^2$ to generate other Pythagorean triples.

   a. Prove the polynomial identity is true by showing that the simplified expressions for the left and right sides are the same.

   b. Use the identity to generate the Pythagorean triple when $x = 6$ and $y = 5$.

   c. Verify that your answer in part (b) satisfies $a^2 + b^2 = c^2$.

Maintaining Mathematical Proficiency
Reviewing what you learned in previous grades and lessons

Perform the operation. Write the answer in standard form. *(Section 4.2)*

66. $(3 - 2i) + (5 + 9i)$

67. $(12 + 3i) - (7 - 8i)$

68. $(7i)(-3i)$

69. $(4 + i)(2 - i)$